Sixth form induction homework

You should complete this work over the summer holiday and bring it to your first maths lesson in September. The work should be done on a separate piece of paper and all working should be shown clearly. Do your best to answer all the questions and use resources such as www.corbettmaths.com and mymaths if you get stuck.

1  Simplify these expressions.
   a  \( \frac{x^3 \times x^4}{x^2} \)  
   (1 mark)
   b  \((2x^3)^4\)  
   (1 mark)
   c  \(\frac{9x^4}{(27x^{-2})^\frac{1}{2}}\)  
   (3 marks)

2  Solve  \(2x^2 \times 4x^4 = 512\)  
   (2 marks)

3  Find the value of \(x\).
   \(x^{-\frac{4}{3}} = \frac{1}{256}\)  
   (2 marks)

4  a  Write \(\sqrt{240}\) in the form \(a\sqrt{15}\), where \(a\) is an integer.  
   (1 mark)
   b  Expand and simplify \((2 - \sqrt{3})(5 + 2\sqrt{3})\).  
   (2 marks)
   c  Simplify \(\frac{2 + \sqrt{5}}{3 - \sqrt{5}}\) giving your answer in the form \(a + b\sqrt{c}\), where \(a\), \(b\) and \(c\) are rational numbers.  
   (3 marks)

5  The area of a triangle is given as \((7 + 3\sqrt{3})\text{ cm}^2\).

   The base of the triangle is \((5 - \sqrt{3})\text{ cm}\), and the perpendicular height is \((p + q\sqrt{3})\text{ cm}\).

   Find the values of \(p\) and \(q\).  
   (4 marks)

6  Expand and simplify these expressions.
   a  \(3(x - 2y)\)  
   (1 mark)
   b  \((2x - 3)(3x + 5)\)  
   (2 marks)
   c  \((x - 2)^2(x + 5)\)  
   (3 marks)
7 Fully factorise these expressions.
   a $2xy - 4x$ (1 mark)
   b $x^2 + 2x - 3$ (1 mark)

8 Solve these equations.
   a $3x - 7 = 17$ (1 mark)
   b $x^2 - 6x + 5 = 0$ (2 marks)
   c $2x^2 - 5x + 1 = 0$ (2 marks)

9 Solve these pairs of simultaneous equations.
   a $2x + y = 7$
      $3x - y = 8$ (3 marks)
   b $y = 3x - 1$
      $3y = 6x + 1$ (3 marks)
   c $2x - y = 9$
      $x^2 + y^2 = 17$ (4 marks)

10 Solve these inequalities.
    a $7x - 6 \leq 8$ (1 mark)
    b $3x + 2 \geq 7x - 4$ (2 marks)
    c $x^2 + 12x - 28 > 0$ (2 marks)

11 The function $f$ is defined as $f(x) = 5x + 2$.
    Find the value of $f(-4)$.
    (1 mark)

12 A triangle $ABC$ has side lengths $AB = 10$ cm, $BC = 15$ cm and $AC = 8$ cm.
   a Find the size of the largest angle, giving your answer to 2 decimal places.
      (3 marks)
   b Find the area of the triangle, giving your answer to 2 decimal places.
      (2 marks)

13 a Sketch the graph of $y = \cos x$ for $-180 \leq x \leq 360^\circ$, showing the points where
    the graph cuts the axes. (2 marks)
   b Hence find the exact values of $x$ in the interval $-180 \leq x \leq 360^\circ$ for which
    $\cos x = -\frac{\sqrt{3}}{2}$ (3 marks)